Investigating the Proton Geometry in the Dipole Model

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BNL 10/13/16

Ongoing work together with

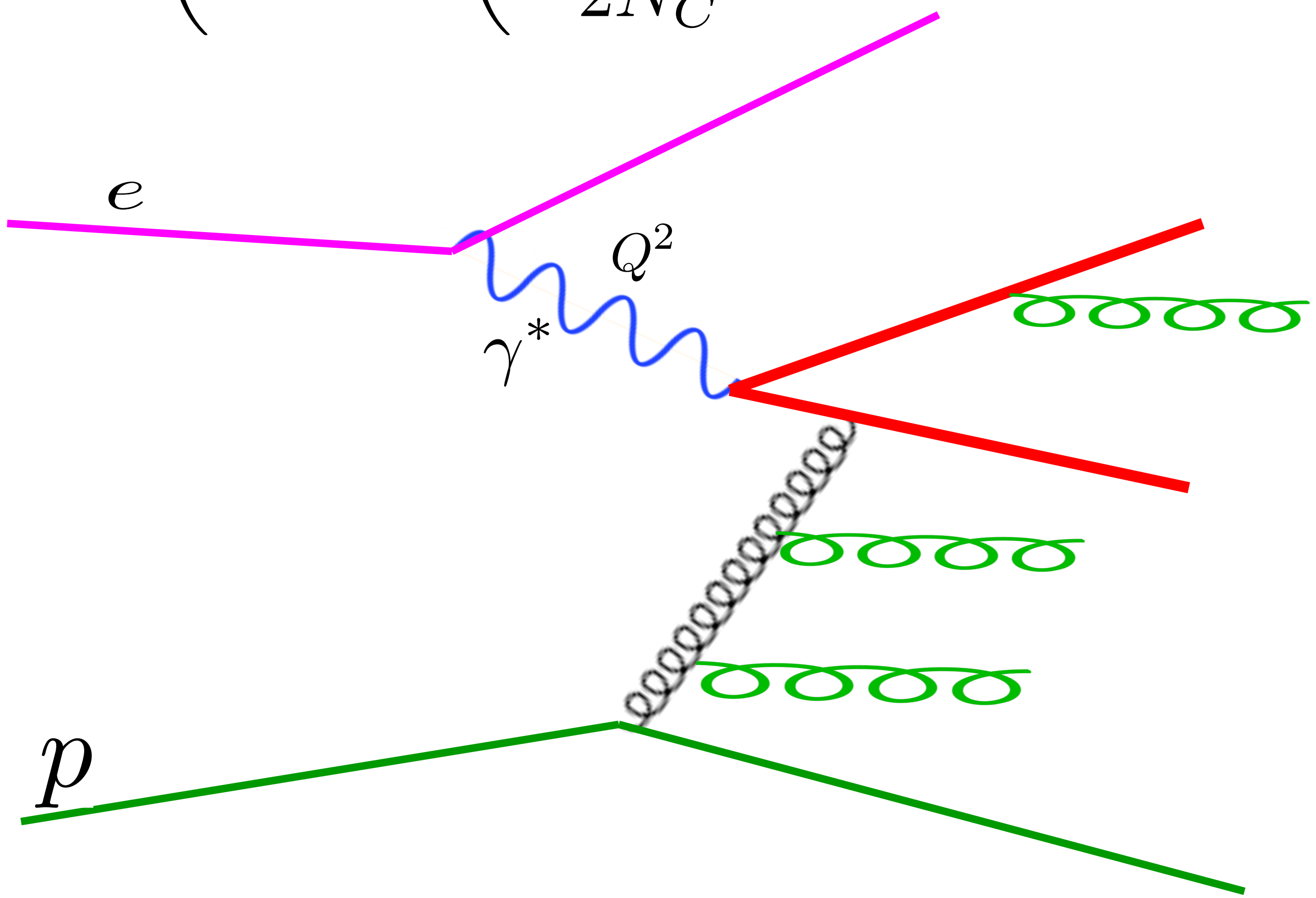
Sakshi Nijhawan and
Bharath Sambasivam

The Dipole Model

Exclusive diffractive processes at HERA within the dipole picture - Kowalski, H. et al. Phys.Rev. D74 (2006) 074016 hep-ph/0606272 DESY-06-095

$$\sigma_{L,T}^{\gamma^* p} = \int \mathrm{d}^2\vec{r} \int_0^1 \mathrm{d}z \left|\Psi_{L,T}^{\gamma^*}\right|^2 \int \mathrm{d}^2\vec{b} \frac{\mathrm{d}\sigma_{\mathrm{dip}}}{\mathrm{d}^2\vec{b}}$$

$$\frac{\mathrm{d}\sigma_{\mathrm{dip}}}{\mathrm{d}^2\vec{b}} = 2\left(1 - \exp\left(-\frac{\pi^2}{2N_C} r^2 \alpha_S(\mu^2) x g(x, \mu^2) T_p(\vec{b})\right)\right)$$



The Dipole Model

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Parameters:

$$\mu^2 = \mu_0^2 + \frac{2}{r^2} \qquad T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{B_G}}$$

$$xg(x,\mu_0^2) = A_g\, x^{-\lambda_g}\, (1-x)^{5.6}$$

$$\text{Parameters to fit : } \underbrace{\mu_0^2,\ A_g,\ \lambda_g}_{\textbf{Use DIS data}},\ B_G$$

Diffraction

The Dipole Model

Rezaeian, Siddikov, Van de Klundert, Venugopalan
Phys.Rev. D87 (2013) no.3, 034002

Data	B_G/GeV^2	$m_{u,d,s}/\mathrm{GeV}$	m_c/GeV	μ_0^2/GeV^2	A_g	λ_g	$\chi^2/\mathrm{d.o.f.}$
σ_r	4	≈ 0	1.27	1.51	2.308	0.058	$298.89/259 = 1.15$
σ_r	4	≈ 0	1.4	1.428	2.373	0.052	$316.61/259 = 1.22$

Parameters:

$$\mu^2 = \mu_0^2 + \frac{2}{r^2} \qquad T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

$$xg(x, \mu_0^2) = A_g\, x^{-\lambda_g}\, (1-x)^{5.6}$$

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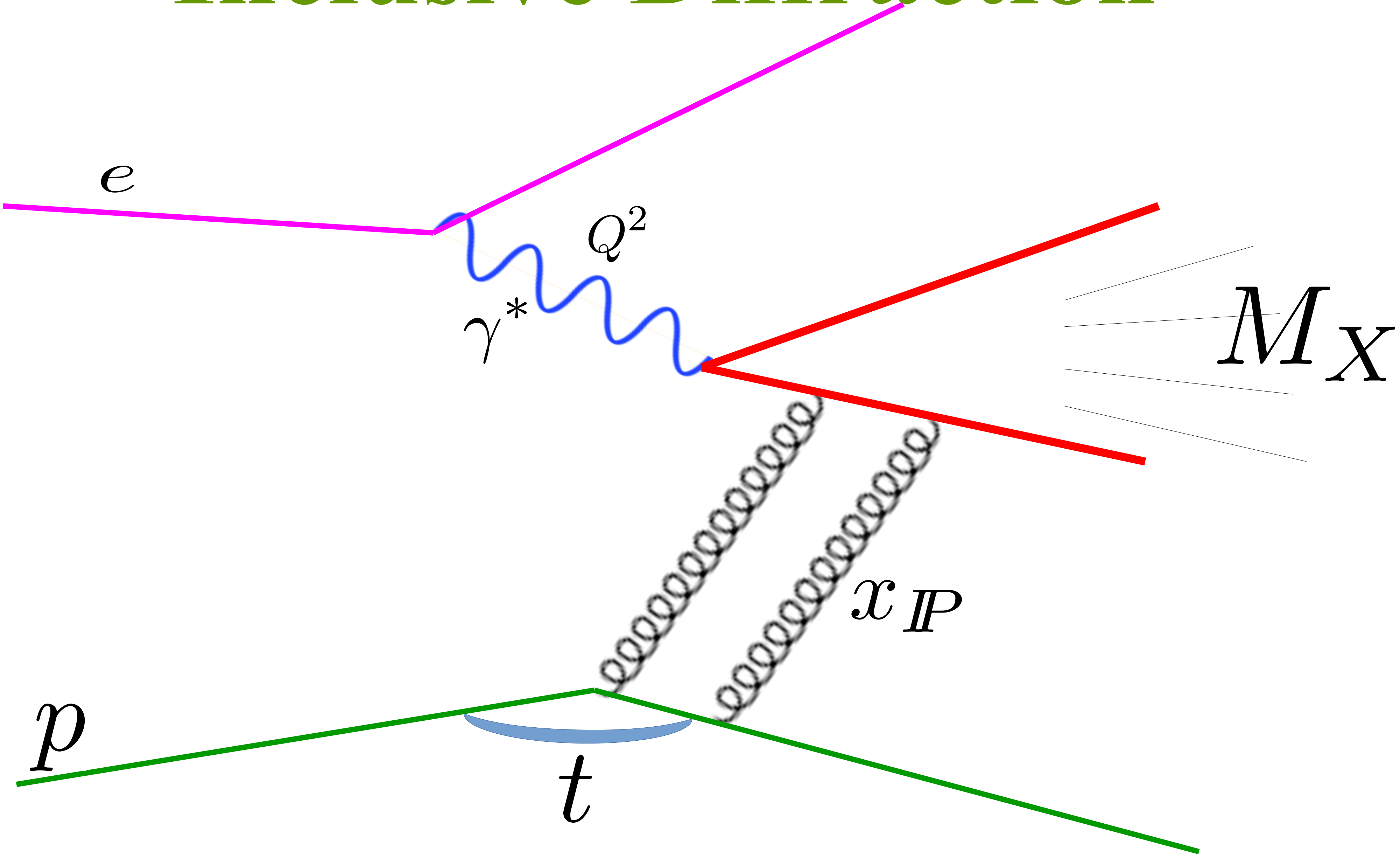
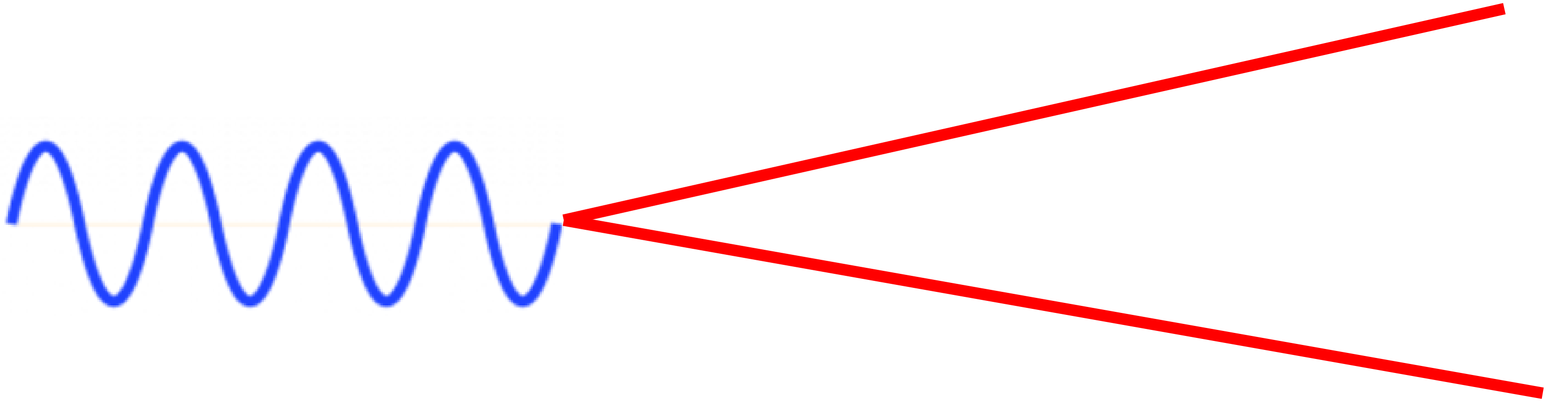
Diffraction

Inclusive Diffraction

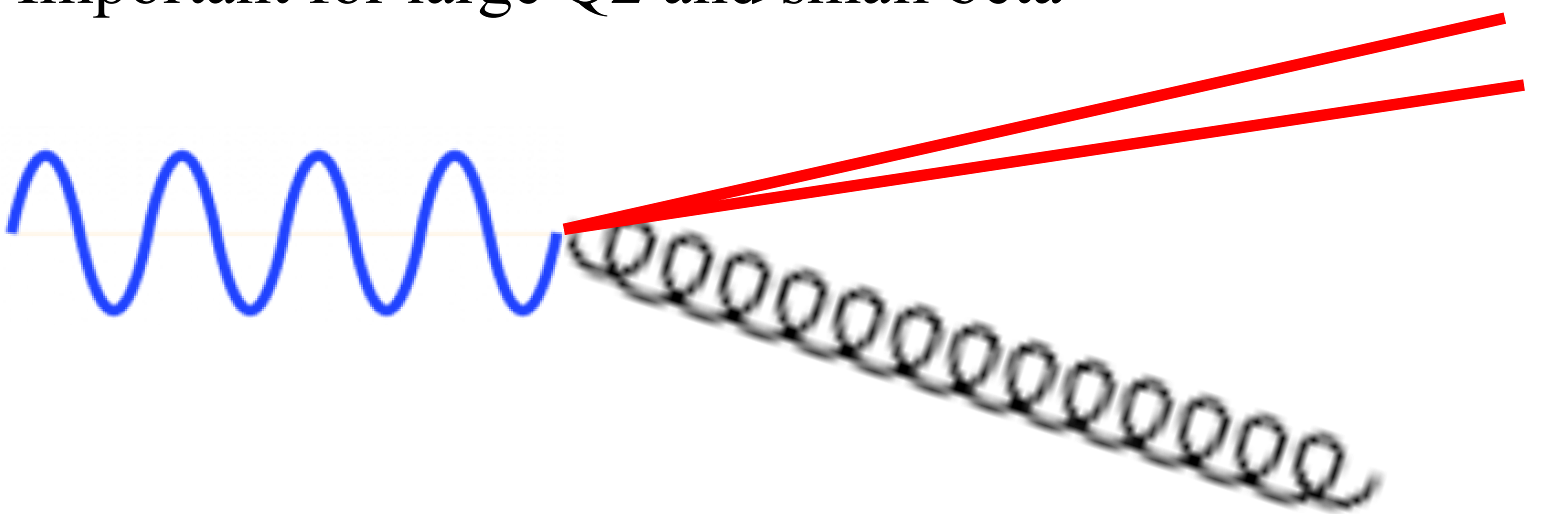


The *qq*-dipole



The *qqg*-dipole

Important for large Q2 and small beta



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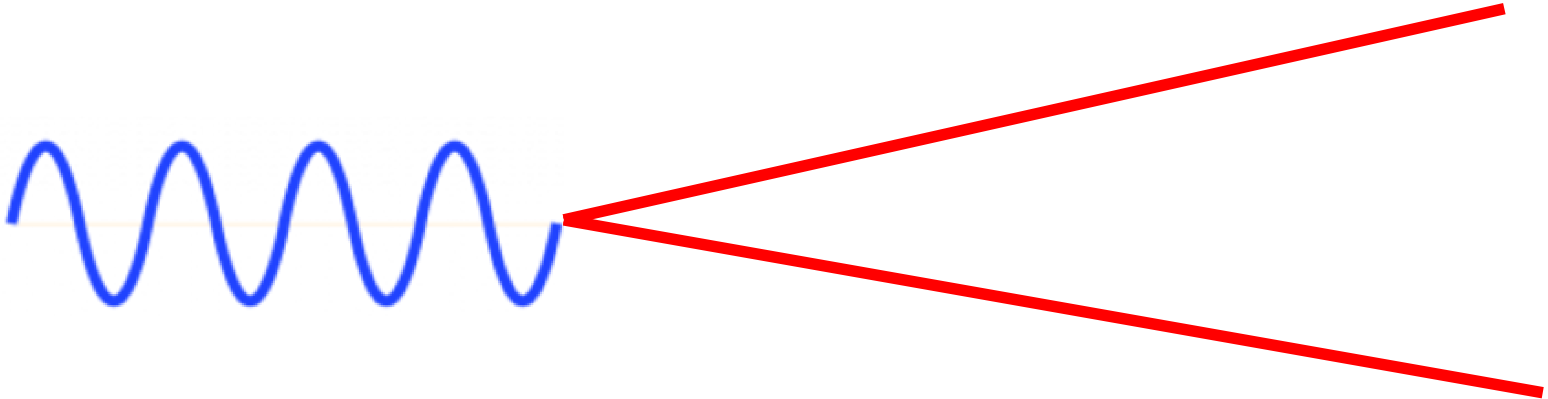

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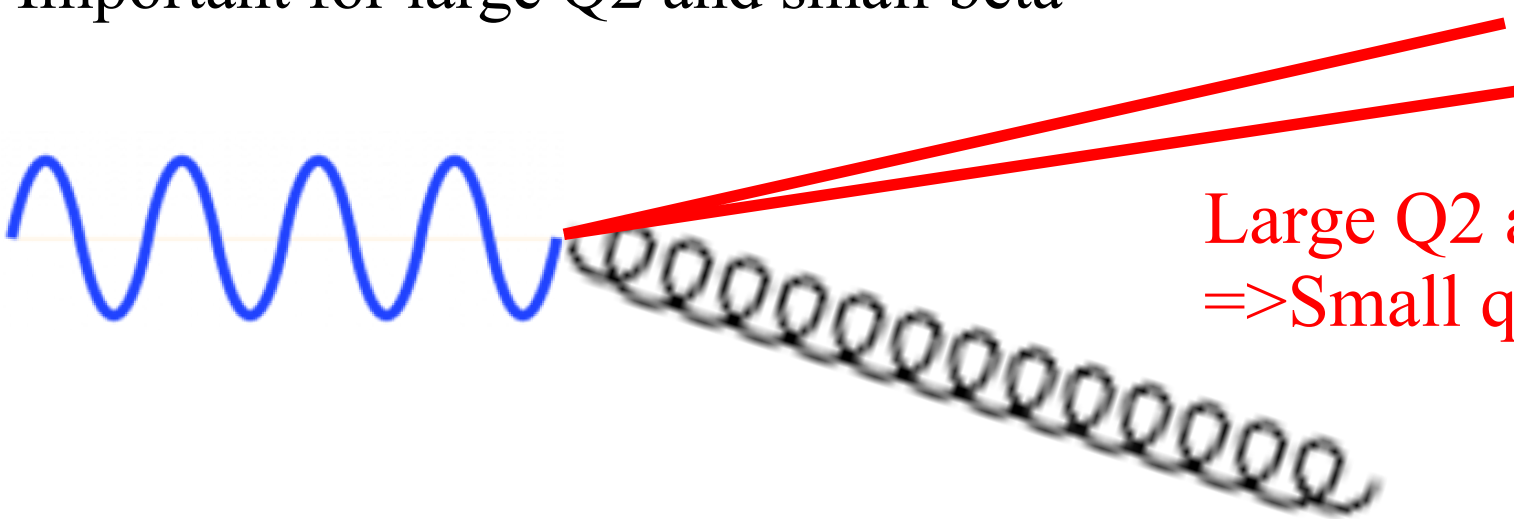

Large Q2 approximation
=>Small qq-dipole

The $q\bar{q}$-dipole

$$\frac{\mathrm{d}\sigma_{\mathrm{dip}}}{\mathrm{d}^2\vec{b}} = 2\left(1 - \exp\left(-\frac{\pi^2}{2N_C} r^2 \alpha_S(\mu^2) x g(x,\mu^2) T_p(\vec{b})\right)\right)$$

$$\mu^2 = \mu_0^2 + \frac{2}{r^2}$$

$$\Phi_n(\beta, Q^2, x_{I\!P}, z) = \int \mathrm{d}^2\mathbf{b} \left| \int \mathrm{d}r\; r K_n(\epsilon r) J_n(\kappa r) \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}}(\mathbf{b}, r, x_{I\!P}) \right|^2$$

$$\frac{\mathrm{d}\sigma_T^{\gamma * p \to Xp}}{\mathrm{d}\beta} = \frac{N_C Q^2 \alpha_{\mathrm{em}}}{4\pi\beta^2} \sum_f e_f^2 \int_{z_0}^{1/2} \mathrm{d}z\; z(1-z) \left[\epsilon^2(z^2 + (1-z)^2)\Phi_1 + m_f^2 \Phi_0\right]$$

$$\frac{\mathrm{d}\sigma_L^{\gamma * p \to Xp}}{\mathrm{d}\beta} = \frac{N_C Q^4 \alpha_{\mathrm{em}}}{\pi\beta^2} \sum_f e_f^2 \int_{z_0}^{1/2} \mathrm{d}z\; z^3(1-z)^3 \Phi_0$$

Nuclear enhancement and suppression of diffractive structure functions at high energies - Kowalski, H. et al. Phys.Rev. C78 (2008) 045201 arXiv:0805.4071 [hep-ph]

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$$\mu^2 = \mu_0^2 + \frac{2}{r^2}$$

$$x_{\mathbb{P}} F_{T,q\bar{q}g}^{\mathrm{D\,(GBW)}}(x_{\mathbb{P}}, \beta, Q^2) =$$

$$\frac{\alpha_s \beta}{8\pi^4} \sum_f e_f^2 \int d^2\mathbf{b}_T \int_0^{Q^2} dk^2 \int_\beta^1 dz \Bigg\{$$

$$k^4 \ln\frac{Q^2}{k^2}\left[\left(1 - \frac{\beta}{z}\right)^2 + \left(\frac{\beta}{z}\right)^2\right]$$

$$\times \left[\int_0^\infty dr\, r \frac{d\tilde{\sigma}_{\mathrm{dip}}}{d^2\mathbf{b}_T}(\mathbf{b}_T, \mathbf{r}_T, x_{\mathbb{P}})\right.$$

$$\left. K_2(\sqrt{z} kr) J_2(\sqrt{1-z} kr)\right]^2 \Bigg\}$$

Inconsistent coupling

Nuclear enhancement and suppression of diffractive structure functions at high energies - Kowalski, H. et al. Phys.Rev. C78 (2008) 045201 arXiv:0805.4071 [hep-ph]

The qqg-dipole

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$$x_{I\!P} F_T^D(x_{I\!P},\beta,Q^2) = x_{I\!P} F_{T,q\bar{q}}(x_{I\!P},\beta,Q^2) + x_{I\!P} F_{T,q\bar{q}g}(x_{I\!P},\beta,Q^2)$$

$$x_{I\!P} F_{T,q\bar{q}g}(x_{I\!P},\beta,Q^2) = x_{I\!P} F_{T,q\bar{q}g}^{\mathrm{large}Q^2}(x_{I\!P},\beta,Q^2) \times \frac{x_{I\!P} F_{T,q\bar{q}g}^{\mathrm{small}\beta}(x_{I\!P},Q^2)}{x_{I\!P} F_{T,q\bar{q}g}^{\mathrm{large}Q^2}(x_{I\!P},\beta=0,Q^2)}$$

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$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

For small dipoles (large Q2) and large t:

$$B_G \approx B_D$$

$$\frac{\mathrm{d}\sigma_{T,L}^{\gamma^* p}}{\mathrm{d}t} \approx e^{-B_D|t|}$$

The

$$\frac{\mathrm{d}\sigma_{\mathrm{dip}}}{\mathrm{d}^2\vec{b}} = 2\left(1 - \exp\left(\right.\right.$$

$$\mu^2 =$$

$$\Phi_n(\beta, Q^2, x_{I\!P}, z) = \int \mathrm{d}^2$$

$$I\!P)\Big|^2$$

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Rezaeian, Siddikov, Van de Klundert, Venugopalan
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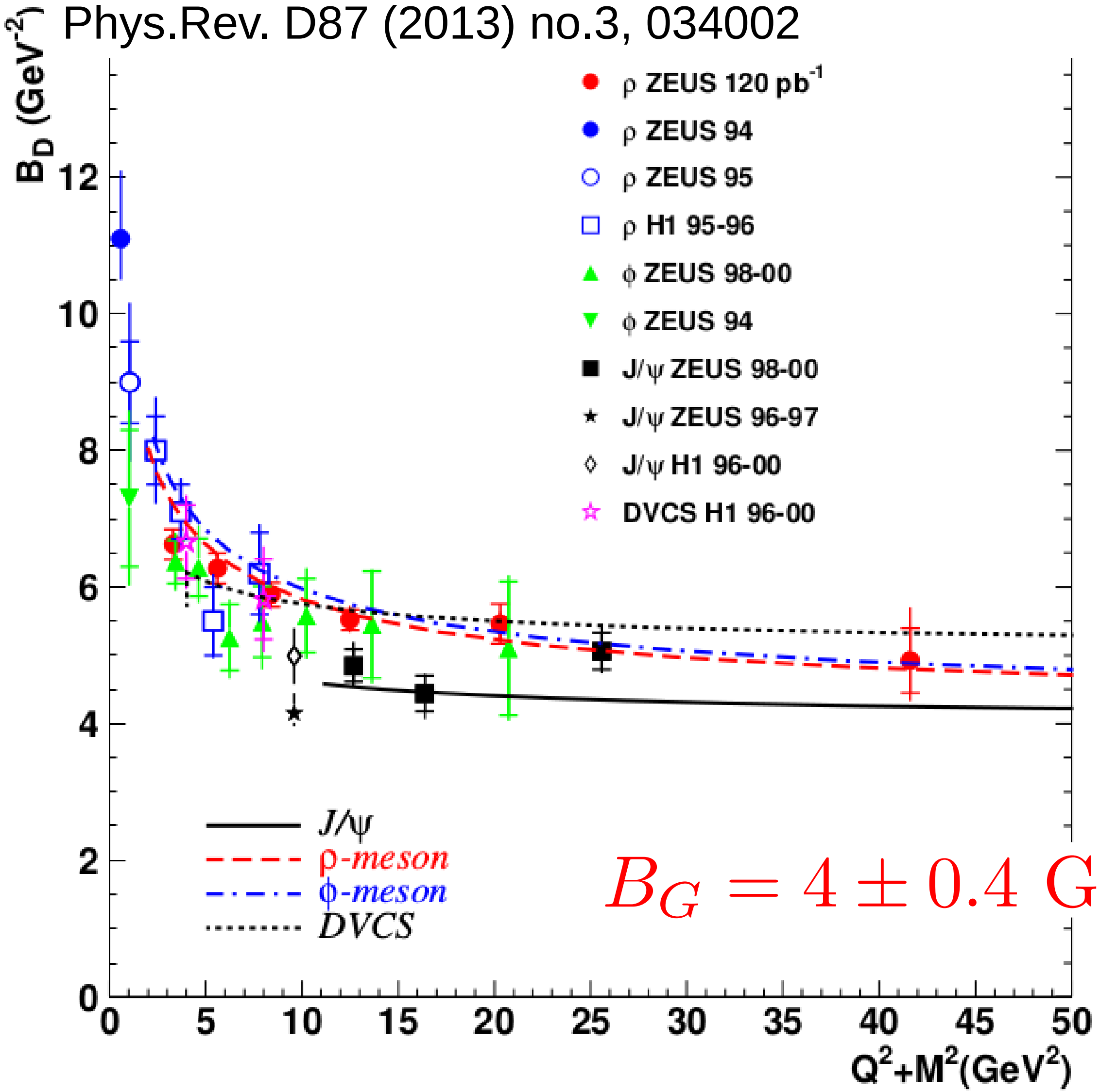

$$B_G = 4 \pm 0.4\ \mathrm{GeV}^{-2}$$

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$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}} \qquad \left(\int |T_p(b)|^2 \, d^2\vec{b} = \frac{1}{2B_G}\right)$$

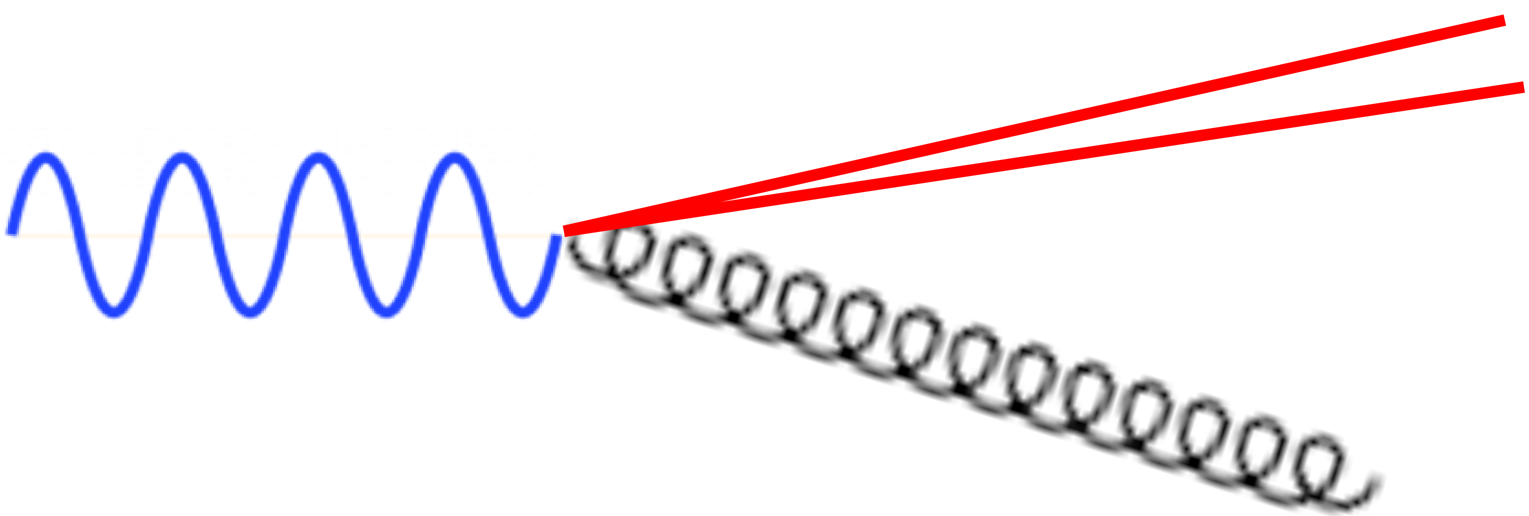
Possibility to use inclusive diffraction data for fit!

The *qqg*-coupling

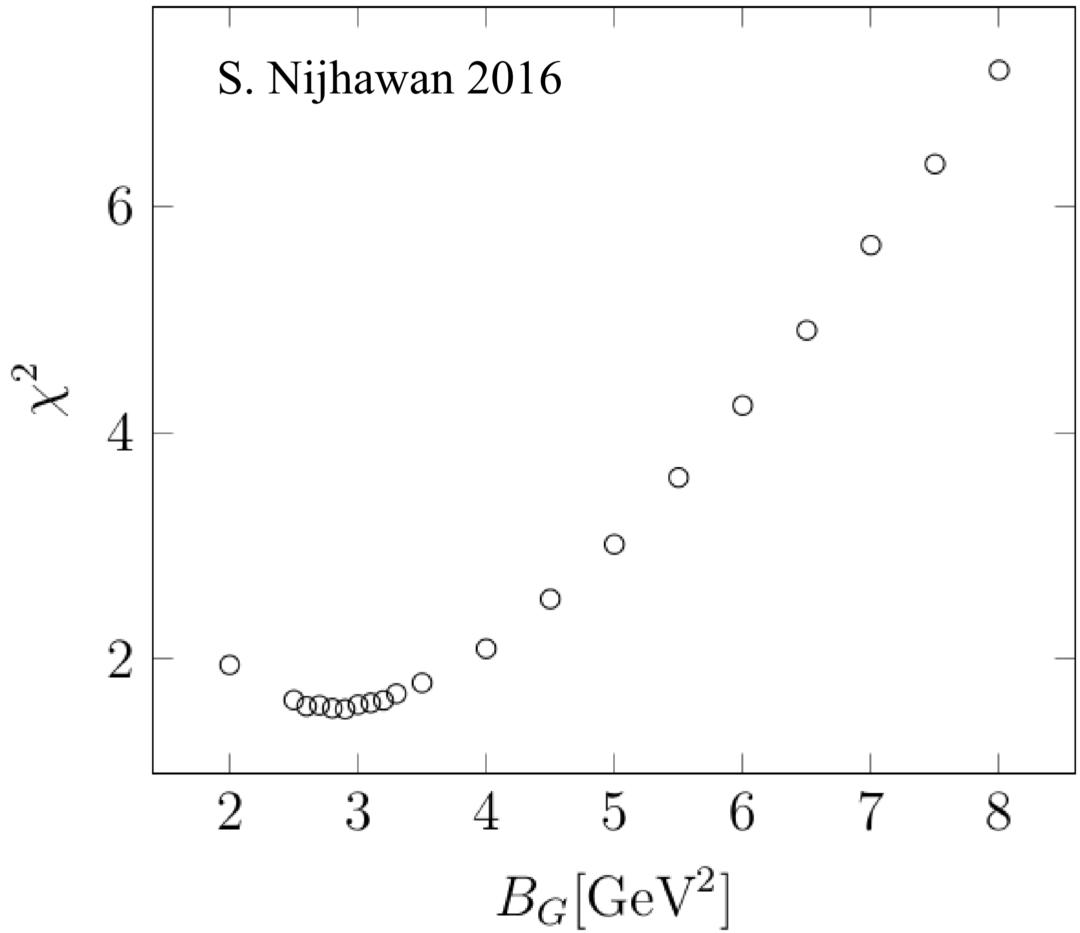
$\alpha_S(\mu^2) \qquad \mu^2 = \mu_0^2 + \frac{2}{r^2}$

Assume large Q2 limit, such that $r^2 = \frac{1}{Q^2}$

Had to set $\alpha_S(M_Z) = 0.091$ for reasonable fit
world average: 0.1184



Fit results



χ^2	dof	χ^2/dof	$B_G(\mathrm{GeV}^{-2})$
124.7	80	1.5	2.9

"Combined inclusive diffractive cross sections measured with forward proton spectrometers in deep inelastic ep scattering at HERA". In: Eur. Phys. J. C72 (2012), p. 2175. doi: 10.1140/epjc/s10052-012-2175-y. arXiv: 1207.4864 [hep-ex]

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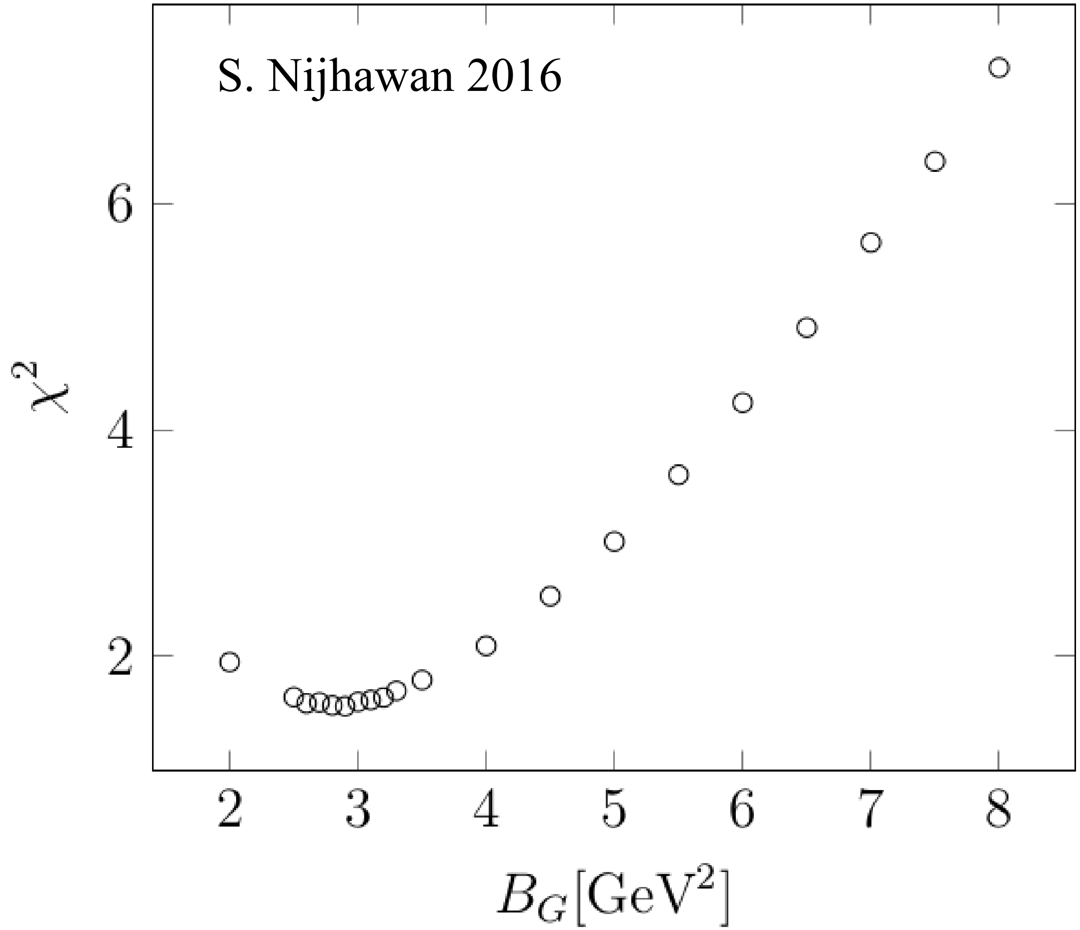

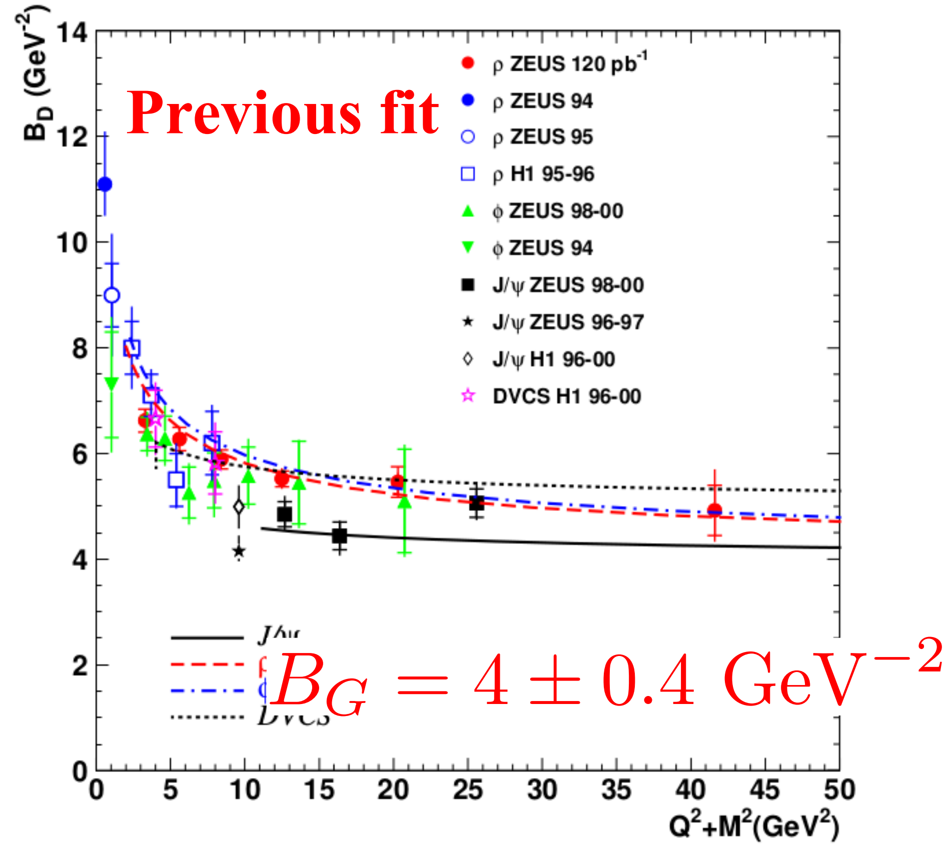

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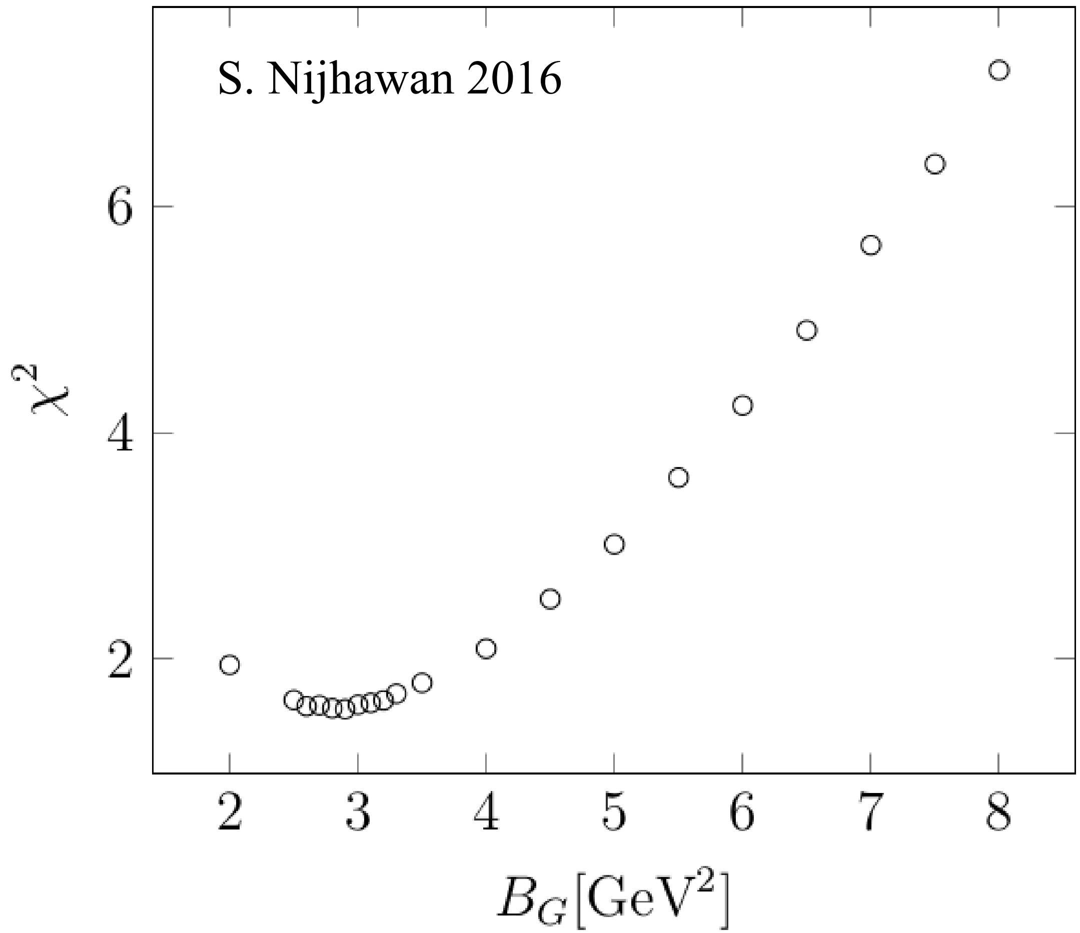

Can we find consistency with different proton shape?

χ^2	dof	χ^2/dof	$B_G(\mathrm{GeV}^{-2})$
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Fit results

Fit in different ranges in Q2 and x

Expectation: Small Q2 and small x would yield larger proton

This is not seen in the fit.

Kinematic Range, $Q^2[\mathrm{GeV}^2]$	χ^2	dof	χ^2/dof	$B_G(\mathrm{GeV}^{-2})$
$Q^2 < 9$	74.0	45	1.6	2.9
$9 < Q^2 < 45$	31.3	18	1.7	2.7
$Q^2 > 45$	17.7	17	1.0	2.9

S. Nijhawan 2016

$Q^2[\mathrm{GeV}^2]$ range	x range	χ^2	dof	χ^2/dof	$B_G(\mathrm{GeV}^{-2})$
$Q^2 < 9$	x<1e-3	10.1	11	0.9	2.7
$Q^2 > 45$	x>1e-3	17.5	16	1.0	2.9

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Summary

We have fitted the proton size parameter to inclusive diffraction data.

The result is inconsistent with fit to exclusive diffraction data

This difference may be due to the shape of the proton not being Gaussian.

Next steps

Include real part corrections to the amplitude.

Modify the strong coupling to include a parameter in qqg

Include exclusive diffraction into the fit?
(Possible problems with Skewedness corrections)

Try different shape functions to find consistent values of variance and normalisation

Small beta corrections

$$x_{\mathbb{P}} F_{T,q\bar{q}g}^{\mathrm{D\ (MS)}}(x_{\mathbb{P}}, \beta = 0, Q^2) = \frac{C_{\mathrm{F}} \alpha_{\mathrm{s}} Q^2}{4\pi^4 \alpha_{\mathrm{em}}} \int \mathrm{d}^2 \mathbf{r}_T \int_0^1 \mathrm{d}z$$

$$\left| \Psi_T^{\gamma^*}(r, Q, z) \right|^2 \int \mathrm{d}^2 \mathbf{b}_T A(r, x_{\mathbb{P}}, \mathbf{b}_T), \quad (1$$

$$A(r, x_{\mathbb{P}}, \mathbf{b}_T) = \int \mathrm{d}^2 \mathbf{r}_T{}' \frac{\mathbf{r}_T{}^2}{\mathbf{r}_T{}'^2 (\mathbf{r}_T - \mathbf{r}_T{}')^2} \Big[\mathcal{N}(\mathbf{r}_T{}')$$

$$+ \mathcal{N}(\mathbf{r}_T - \mathbf{r}_T{}') - \mathcal{N}(\mathbf{r}_T) - \mathcal{N}(\mathbf{r}_T{}') \mathcal{N}(\mathbf{r}_T - \mathbf{r}_T{}') \Big]^2$$

$$x_{I\!P} F_{T,q\bar{q}g}(x_{I\!P}, \beta, Q^2) = \mathrm{x}_{I\!P} F_{T,q\bar{q}g}^{\mathrm{large}Q^2}(x_{I\!P,\beta,Q^2}) \times \frac{x_{I\!P} F_{T,q\bar{q}g}^{\mathrm{small}\beta}(x_{I\!P}, Q^2)}{x_{I\!P} F_{T,q\bar{q}g}^{\mathrm{large}Q^2}(x_{I\!P}, \beta=0, Q^2)}$$

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